DETERMINATION OF LOSSES OF EXERGY FROM REGENERATIVE HEAT TRANSFER IN THE CYLINDER OF A PISTON EXPANSION COOLER

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The losses of exergy due to regenerative heat transfer in a piston expansion cooler are determined analytically. The dependence of the heat flux, taking part in the regenerative heat transfer, on the thermophysical properties of the materials of the cylinder group is analyzed.

In the investigation of piston expansion coolers a large attention is devoted to the study of regenerative heat transfer [1-5]. However, many problems related to regenerative heat transfer are still not sufficiently clear.

As yet there are no methods for an analytic computation of losses due to regenerative heat transfer; the effect of the thermophysical properties of the constructional materials on the heat flux Q_p, taking part in regenerative heat transfer, has also been studied inadequately.

An analytic determination of losses and the effect of some factors on Q_p are considered below.*

The heat flux Q_p, developing between the expanding gas in the machine and the walls of the cylinder results in a loss of exergy [7]; for an elementary segment the instantaneous value of this loss can be written in the following way:

$$\delta d = \Delta\tau_e \delta Q_p, \tag{1}$$

where

$$\Delta\tau_e = \frac{T_1 - T_{\text{amb}}}{T_1} - \frac{T_2 - T_{\text{amb}}}{T_2}.$$

The total loss during one period of the machine cycle is

$$D = \int_0^{t_0} \Delta\tau_e \delta Q_p. \tag{2}$$

It is known [3] that the temperature of a gas in the cylinder of a piston expansion cooler changes according to a law close to harmonic. If the cosine law of variation of the temperature of the gas in the cylinder is taken as the first approximation, then using known assumptions [1] the instantaneous values of T_1, T_2, and δQ_p can be written in the form

1) temperature of the gas

$$T_1 = T_{\text{av}} + T_{\text{M}} \cos \frac{2\pi t}{t_0}, \tag{3}$$

2) temperature of the wall

$$T_2 = T_{\text{av}} + T_{\text{M}} \eta_0 \cos\left(2\pi \frac{t}{t_0} - \varepsilon_0\right), \tag{4}$$

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where $\eta_0 = \sqrt{1/(1 + 2[\sqrt{\pi/h^2at_0}] + 2\,[\pi/h^2at_0])}$ is a factor showing how many times the amplitude of oscillations of the temperature at the surface of the wall is smaller than the amplitude of oscillation of the gas temperature; $\varepsilon_0 = \arctan 1/(1 + [\sqrt{h^2at_0/\pi}])$ is a quantity that takes account of the phase shift of the temperature oscillations of the wall and the gas [1].

3) the amount of heat given out from the gas to the wall (or the reverse) during the time dt,

$$dQ_p = \bar{\alpha} F (T_1 - T_2)\, dT. \tag{5}$$

Neglecting the ratio of the crank radius to the length of the connecting rod of the expansion cooler, the expression for F has the form

$$F = 2F_{c1} + \frac{F_{cm}}{2}\left(1 - \cos 2\pi \frac{t}{t_0}\right). \tag{5a}$$

Substituting the values of $\Delta\tau_e$, T_1, T_2, and dQ_p into Eq. (2) we have

$$D = \bar{\alpha} T_{amb} \int_0^{t_0} \left[\frac{1}{T_{av} + T_M \eta_0 \cos\left(2\pi \frac{t}{t_0} - \varepsilon_0\right)} - \frac{1}{T_{av} + T_M \cos 2\pi \frac{t}{t_0}}\right]$$

$$\times \left[2F_{c1} + \frac{F_{cm}}{2}\left(1 - \cos 2\pi \frac{t}{t_0}\right)\right] \left\{T_M \left[\cos 2\pi \frac{t}{t_0} - \eta_0 \cos\left(2\pi \frac{t}{t_0} - \varepsilon_0\right)\right]\right\} dt. \tag{6}$$

In integrating expression (6) we assume that

$$\frac{T_{av}}{T_M} \gg \eta_0 \cos\left(2\pi \frac{t}{t_0} - \varepsilon_0\right); \quad \frac{T_{av}}{T_M} \gg \cos 2\pi \frac{t}{t_0}.$$

Then Eq. (6) becomes

$$D = \frac{\bar{\alpha} T_{amb} T_M t_0}{4T_{av}^2}(F_{cm} + 4F_{c1})\left[1 - 2\eta_0 \cos \varepsilon_0 + \eta_0^2 (\pi - \varepsilon_0)\right]. \tag{7}$$

The values of D, computed from Eq. (7) and obtained from thermodynamic analysis of losses using measurements of internal parameters of the gas in an expansion cooler [4], comprise 5 and 5.77 kJ/kg, respectively. Therefore, the final results for D, obtained with the assumption of cosine law of motion of the piston, are close to the experimental values.

Expression (7) enables one to analyze the effect of some factors on the loss due to regenerative heat transfer. Thus, the value of D is directly proportional to α, t_0, T_M and inversely proportional to T_{av}^2.

The dependence of D on the thermophysical properties of the materials of the cylinder group is appreciably more complex. It can be analyzed most simply through the variation of Q_p.

The heat flux, taking part in regenerative heat transfer, can be calculated from the equations given in [1] with well known assumptions:

$$Q_p = \sqrt{\frac{2}{\pi}} \sqrt{\lambda c \gamma} \sqrt{t_0}\, F T_M \eta_0. \tag{8}$$

In order to determine the effect of the thermophysical properties of the material of the cylinder group on Q_p we write Eq. (8) (for F = 1) in the following form:

$$q_p = B \sqrt{\frac{x}{1 + 2\sqrt{x} + 2x}}, \tag{9}$$

where

$$B = \frac{T_M \sqrt{2}}{A}\, t\bar{\alpha}; \quad x = A\lambda c\gamma; \quad A = \frac{\pi}{t\bar{\alpha}^2}.$$

An investigation of the function $q_p = f(x)$ for constant coefficients A and B shows that it increases continuously from zero and has the limit (for $x \to \infty$) $B/\sqrt{2}$:

$$\lim_{\lambda c\gamma \to \infty} q_p = \frac{B}{\sqrt{2}} = \frac{T_M}{\pi}\, t^2 \bar{\alpha}^3. \tag{10}$$

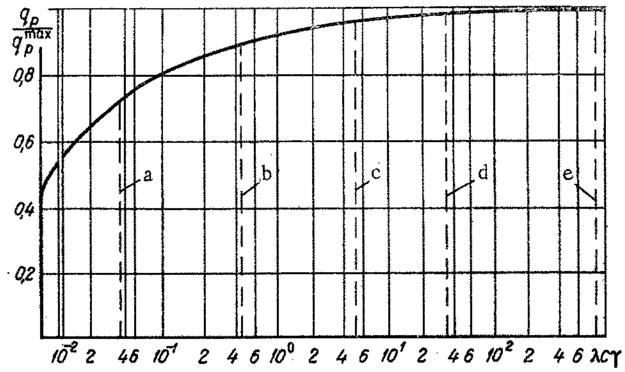

Fig. 1. Dependence of heat flux on the thermophysical properties of the materials of the cylinder group for different temperatures ($\lambda c \gamma$, $kJ^2/sec \cdot m^4 \cdot deg^2$): a) steel 1Kh18N9T at T = 20°K; b) Textolite at T = 250°K; c) copper M3 at T = 20°K; d) steel 1Kh18N9T at T = 250°K; e) copper M3 at T = 250°K.

The dependence of the quantity q_p on the complex $\lambda c \gamma$ is shown in Fig. 1. It is evident from the figure that q_p changes insignificantly on changing $\lambda c \gamma$ from 100 to ∞ (roughly by 10% of the maximum value of q_p). This is accounted for by the fact that simultaneously with the increase in the accumulation coefficient ($\lambda c \gamma$) the amplitude $T_M \eta_0$ of the temperature oscillations at the surface of the walls decreases, which in turn leads to a slowing down of the growth of q_p. It is also seen from the figure that a replacement of one material by another differing in its thermophysical properties (the complex $\lambda c \gamma$) by an order of magnitude does not cause a significant variation in q_p and, hence, has a small effect on the efficiency of the machine.

The curve in Fig. 1 is plotted for the computed quantities corresponding to the values of q_p for Textolite, copper, and stainless steel (1Kh18N9T) at temperatures of 250 and 20°K. The maximum difference in the values for one and the same temperature (250°K) does not exceed 10%.

Experiments carried out on expansion cooler of MÉI [4] confirm these conclusions. The replacement of the Textolite cylinder by the copper cylinder (the remaining parameters unchanged) leads to a decrease of the adiabatic efficiency of the machine by 3.5%.*

A thermodynamic analysis of the operation of this expansion cooler, carried out from the measurements of the gas parameters in the cylinder of the machine, shows that this change in the efficiency corresponds to an increase of q_p by 9.7-10.8%.

A substantial decrease in q_p can be obtained only by using materials for which the complex $\lambda c \gamma$ is smaller than $1.5 \cdot 10^{-1}$. From this point of view it is advisable to line the cylinder and the piston by a material of the type of polysterene plastic for which $\lambda c \gamma = 1.4 \cdot 10^{-3}$ [6] (at T = 293°K). However, in this case constructional difficulties appear, mainly due to the increase in the linear expansion coefficient.

NOTATION

T_{amb}	is the ambient temperature;
T_1 and T_2	are the instantaneous values of the temperature of the gas and the wall of the cylinder;
T_{av}	is the average temperature of the gas;
T_M	is the amplitude of oscillation of the gas temperature;
t	is the instantaneous value of time;
$\bar{\alpha}$	is the conventional heat transfer coefficient from gas to cylinder wall;
λ, c, γ	are the thermal conductivity, heat capacity, and density of the material of the cylinder wall and piston;
F	is the heat transfer surface;
F_{cm}	is the maximum lateral surface of the cylinder;

*The adiabatic efficiency of the expansion cooler with Textolite and copper cylinder was 70.1 and 66.6%, respectively.

F_{cl} is the area of the cylinder lid or the face of the piston head;

a is the thermal conductivity of the material of the cylinder;

h is the relative heat transfer coefficient;

q_p^{max} is the maximum value of specific heat flux.

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